B.Sc. 1st Semester (Hons.) Examination,

November-2014

PHYSICS

Paper-Phy-104

Mathematics-I

Time allowed: 3 hours]

[Maximum marks: 40

Note: Attempt five questions in all, selecting at least two questions from each section. All questions carry equal marks.

Section-I

- 1. (a) Prove that every convergent sequence is bounded but not conversely.
 - (b) State and prove Cauchy's first theorem on limits.
- 2. (a) Prove that $\lim_{n \to \infty} \left(\frac{n^n}{n!} \right)^{1/n} = e$
 - (b) Prove that a monotonically increasing sequence
 < a_n > which is bounded above converges to its
 least upper bount.

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- 3. (a) Prove that every Cauchy's sequence is bounded.

 Is the converse true? Show by an example. 4
 - (b) Discuss the convergence of the sequence $< a_n >$ where $a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + ... + \frac{1}{3^n}$.
- 4. (a) Prove that a positive term series either converges or diverges to $+\infty$.
 - (b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ converges to $\frac{1}{4}$

Section-II

- 5. (a) If $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} V_n$ are two series of positive terms and $\sum_{n=1}^{\infty} u_n$ is divergent and there is positive constant k such that $u_n \le kv_n$, \forall n, then show that $\sum_{n=1}^{\infty} V_n$ is also divergent.
 - (b) Discuss the convergence of the series:

$$\sum_{n=1}^{\infty} \left(\sqrt{n^3 + 1} - \sqrt{n^3} \right)$$

6. (a) Test the convergence of the series:

$$\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \dots, x > 0$$

(b) Discuss the convergence of the series:

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n, \quad x > 0.$$

- 7. (a) State and prove Cauchy's integral test.
 - (b) Test the convergence of the series

$$x^{2} + \frac{2^{2}}{3.4} x^{4} + \frac{2^{2}.4^{2}}{3.4.5.6} x^{6} + \frac{2^{2}.4^{2}.6^{2}}{3.4.5.6.7.8} x^{8} + \dots, \quad x > 0.$$

- 8. (a) Show that every absolutely convergent series is convergent. Is the converse true? Show by an example.
 - (b) Test the convergence and absolute convergence of the series:

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$